ESTIMATION OF PLANT DENSITY USING LINE TRANSECTS

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(*With seven Figures in the Text*)

I. Introduction

The line-transect method for the estimation of the percentage ground cover by different species in a stand is well established in theory and practice as giving a level of precision in the estimate for a given effort which compares very favourably with other methods. This is especially so when marked aggregation of species occurs. The use of transect data for auxiliary measures indicative of plant numbers for each species has been examined by Bauer (1943), using artificial populations. Starting with known numbers of circles of particular sizes and colours he computed the expected relative number of each type of circle contacting the transect (numerical abundance) and also the relative proportions of transects contacted by the various types (frequency). The estimated measures showed good agreement with experimental data obtained by random transects.

The deficiency of 'numerical abundance' as a measure of abundance of a species is obvious, as it is determined both by density, that is the number of plants per unit area, and by the frequency distribution of diameters. Bauer does not consider the inverse problem of estimating density from transect data. The probability of a circle of a particular diameter being cut by a transect at random is proportional to an area surrounding the transect with boundary at radius distance from the transect line and its terminal points, and the expected number of contacts for all plants of the same diameter is proportional to this area and the number of plants in the class. This weighting of the frequencies in different diameter classes by a factor nearly proportional to the diameter is a source of difficulty in considering the inverse problem of estimating density from chords of intersected plants when the frequency distribution of diameters of the sampled population is not known. However, the possibility of measuring density from transect data is a matter of some interest to workers concerned with open grassland and low shrub communities and an examination of the point seems warranted.

II. Estimation of density from chords on the line transect

The theory of the inverse problem will first be examined with respect to a very large population of circles of varying diameter distributed, not necessarily at random, over a plane of considerable extent. Random line transects are used to sample a restricted portion of the plane. Let the relative frequency distribution of the diameters of circles in this restricted portion be $\phi(D)$, where $\int_0^\infty \phi(D)\, dD = 1$. The limits 0 to ∞ for the variable are equivalent to using the smallest and largest diameters of the population. The diameters, in this notation, are regarded as sufficiently numerous to form in effect a continuous frequency distribution.

For the development of the argument it is convenient to introduce at this stage the restriction that only intersections between a transect and a circle which give rise to a full chord or a part chord at one specified end of the transect will be considered. The admissible area within which centres of circles of diameter D may lie to give rise to such chords is indicated by the cross-hatched area in Fig. 1.

The expected number of intersections of circles with diameters in the interval $D \pm \frac{1}{2}dD$ with a randomly placed transect of length L is proportional to $DL\phi(D)\,dD$. Denote this by $f(D)\,dD$ and the constant of proportionality by α. Then $\frac{f(D)}{LD} = \alpha\phi(D)$, and integrating over all diameters

$$\int_0^\infty \frac{f(D)}{LD}\,dD = \alpha,$$

that is,

$$\phi(D) = \frac{f(D)}{D} \Big/ \int_0^\infty \frac{f(D)}{D}\,dD.$$

Suppose the number of circles per unit area is k, where unit area is of course the square unit corresponding to the unit in which D and L are measured, so that there are $k\phi(D)\,dD$ circles in the range of diameters $D \pm \frac{1}{2}dD$ per unit area. Then the area occupied by circles of diameter D in this range is $\frac{1}{4}\pi D^2 k\phi(D)\,dD$, and the total area occupied is $\frac{1}{4}\pi k \int_0^\infty \phi(D)\,D^2 dD$ per unit area, or

$$\tfrac{1}{4}\pi k \frac{\int_0^\infty Df(D)\,dD}{\int_0^\infty \frac{f(D)}{D}\,dD}.$$

This can be equated to the expectation of the sum of the chords intercepted on n random transects each of length L divided by the total length of transect, the chords including the whole and part chords from circles whose centres lie in both the hatched and unhatched areas from Fig. 1. The expectation of the contribution of part chords from circles with centres in the lower unhatched circle is equal to the expectation of the additional length of intersection which would occur if the transect were projected to complete part chords for circles with centres in the hatched circle. That is, there would be no bias in the estimation of basal area if part chords at one end were ignored and part chords at the other end were completed. Denoting the sum of the chords measured by this latter procedure as ΣCh, an unbiased estimate of k is given by

$$\frac{\Sigma Ch}{nL} \frac{1}{\frac{1}{4}\pi \int_0^\infty \phi(D)\,D^2 dD},$$

which may be estimated by

$$\frac{\Sigma Ch}{nL} \frac{\Sigma(1/D)}{\frac{1}{4}\pi \Sigma D}, \qquad (1)$$

where all summations are taken over the circles intersected by the n transects excluding those contributing part chords at one end.

Now random transects orientated in a particular direction and intersecting a circle of diameter D are equally likely to cut the diameter at right angles to the line of transects at any distance from the centre. If the distance measured to the point of intersection from the centre is $\frac{1}{2}x$ and the complete chord of intersection is Ch, then $x^2+Ch^2=D^2$. All values of dx are equally likely and have probability dx/D. Also

$$\frac{Ch\,d(Ch)}{(D^2-Ch^2)^{\frac{1}{2}}}=dx,$$

so that the probability that a complete chord of intersection of a transect with a circle of diameter D will lie in the range $Ch\pm\frac{1}{2}d(Ch)$ is

$$\frac{Ch\,d(Ch)}{D(D^2-Ch^2)^{\frac{1}{2}}}$$

(see Fig. 2). Considering the chords of intersection with circles whose centres are in the hatched area, chords being completed by projection of the transect where necessary, we have

$$\text{Expectation of } 1/Ch=\tfrac{1}{2}\pi(1/D)$$

$$\text{Expectation of } Ch \quad =\tfrac{1}{4}\pi D.$$

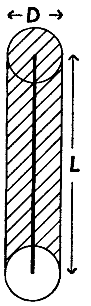


Fig. 1. Area within which the centres of circles of diameter D which contact the transect must lie.

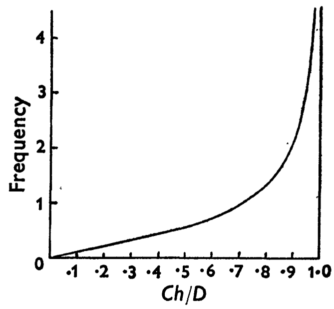


Fig. 2. Frequency distribution of Ch/D.

Substituting in (1) above, k is estimated by

$$\frac{\Sigma Ch}{nL}\,\frac{(2/\pi)\,\Sigma(1/Ch)}{\Sigma Ch} \quad \text{or} \quad \frac{(2/\pi)\,\Sigma(1/Ch)}{nL}. \tag{2}$$

The term $\Sigma(1/Ch)$ is obviously sensitive to errors of measurement for small chords and to irregularities in the boundary so far regarded as circular. With circles the frequency of chord length decreases proportionally with chord length when this is becoming very small, so that very few short chords are contributed from a circle of moderate diameter. If we now consider an irregular boundary such as a scalloped edge, the probability of short chords is considerably increased, and consequently there could be a serious over-estimate of density using the above expression.

With an angular shape such as a triangle the expectation of $1/Ch$ is infinite. The practical importance of these edge irregularities will depend of course on the form of the vegetation and the extent to which contacts at the tips of serrations are ignored. For an experimental distribution discussed later, bias if present seems to be trivial, but this example, although based on pantographic outlines of the base of perennial grasses, probably oversimplifies the field problem in definition of boundary. Alternative procedures which are potentially less sensitive to boundary irregularities will next be considered.

III. Estimation of density from longest chords parallel to transect

The first suggestion is to modify the technique of measurement by measuring the longest chord parallel to the transect for contacted plants with centres in the hatched area instead of the chord of intersection of the plant and the transect.

Since the expectation of Ch is $\frac{1}{4}\pi D$, we derive from (1) that k is estimated by

$$\frac{\Sigma(1/D)}{nL}. \tag{3}$$

As we are now using reciprocals of central chords boundary irregularities will be of less importance than when reciprocals of chords on the line transect are used.

An expression analogous to (3) may be developed to cover the general case of an irregular closed shape. The convention previously adopted for part chords as the ends of transects will be adhered to.

Consider a family of closed curves all of which are undistorted magnifications or reductions of a common shape, and orientated at random in the sampled area. Then if Y is some reference chord, say the longest possible chord, the family can be specified by a relative frequency distribution $\phi(Y)$ which can be estimated from the frequency distribution of intersections of these shapes with n random transects as

$$\phi(Y) = f(Y)/Y \Big/ \int_0^\infty \frac{f(Y)}{Y}\, dY.$$

The integration should strictly be represented by a summation over the values of Y for the finite number of plants intersected by n quadrats where n is extremely large. The area occupied by a shape specified by Y is RY^2, where R is a constant common to all members of the family. The total area covered by a density of k members of this family per unit area is then

$$\frac{kR\int_0^\infty Yf(Y)\, dY}{\int_0^\infty \frac{f(Y)}{Y}\, dY} \quad \text{per unit area of surface.}$$

The expected total length of chord intersected on the n transects is $T\int_0^\infty Yf(Y)\,dY$, where T is the ratio of the expectation of the chord of intersection to the longest chord, this ratio being constant for all values of Y.

For a fixed direction the mean chord of intersection is the ratio of the area, A, to the maximum perpendicular distance, W, between tangents parallel to the transects. The probability of intersection of a transect with a member of the family is proportional to this distance, so that in averaging over all orientations the particular orientation has to be weighted proportional to this distance, i.e. the expectation of the chord of intersection is

$$\frac{\int_0^\pi \frac{A}{W} W\,d\theta}{\int_0^\pi W\,d\theta} = \frac{\pi A}{\int_0^\pi W\,d\theta},$$

where θ is measured in radians and

$$T = \frac{\pi A}{Y \int_0^\pi W\,d\theta}.$$

For convenience it will be specified that T be expressed in terms of Y_0, A_0 and W_0 for a reference shape of particular size.

The estimation of area from intersected chords is then

$$\frac{T}{nL} \int_0^\infty Y f(Y)\,dY.$$

An unbiased estimate of k is

$$\frac{\int_0^\infty \frac{f(Y)}{Y}\,dY}{R \int_0^\infty Y f(Y)\,dY} \; \frac{T \int_0^\infty Y f(Y)\,dY}{nL},$$

which may be estimated by

$$\frac{\Sigma(1/Y)}{nL} \frac{T}{R} = \frac{\Sigma(1/Y)}{nL} \frac{\pi A}{Y_0 \int_0^\pi W_0 d\theta} \frac{Y_0^2}{A_0} = \frac{\Sigma(1/Y)}{nL} \frac{\pi}{\int_0^\pi \frac{W_0}{Y_0} d\theta},$$

where n is now small.

For any given intersection of a transect with a member of the family suppose we measure the longest chord parallel to the direction of the transect. This particular measure is used because it is identifiable. A convention must be adopted on whether only the parts of a chord actually covering the plant are to be accepted. This convention has been used for illustrative examples given later, but if longest chords, counting re-entrants, were used, the procedure would be the same except that the correction factor would also be based on this convention. In Fig. 5 the longest chord parallel to the transect, ignoring re-entrant sections, is indicated. If the convention had been adapted to take the longest chord including any re-entrant section, this chord would have lain slightly to the left of the transect.

For given Y the expectation of the reciprocal of the longest chord D parallel to the transect is

$$\frac{\int_0^\pi \frac{W}{D} d\theta}{\int_0^\pi W\,d\theta}.$$

$$1/Y = \text{mean value of } 1/D \frac{\int_0^\pi \frac{W}{Y} d\theta}{\int_0^\pi \frac{W}{D} d\theta}.$$

$$= \text{mean value of } 1/D \frac{\int_0^\pi \frac{W_0}{Y_0} d\theta}{\int_0^\pi \frac{W_0}{D_0} d\theta}.$$

Therefore k may be estimated by

$$\frac{\Sigma(1/D)}{nL} \frac{\int_0^\pi \frac{W_0}{Y_0} d\theta}{\int_0^\pi \frac{W_0}{D_0} d\theta} \frac{\pi}{\int_0^\pi \frac{W_0}{Y_0} d\theta} = \frac{\Sigma(1/D)}{nL} \frac{\pi}{\int_0^\pi \frac{W_0}{D_0} d\theta} = \frac{\Sigma(1/D)}{nL} \frac{1}{C}, \tag{4}$$

where C is the mean value of the ratio of the maximum perpendicular distance between tangents parallel to the transect to the longest chord parallel to the transect giving equal weight to all angles of orientation.

In the limit with the plants of a species having a variety of shapes, the estimate of density of a particular shape is $\frac{\Sigma(1/D)}{CnL}$, and the estimate of density of plants of all shapes is $\frac{\Sigma\Sigma(1/D)}{\bar{C}nL}$, where $1/\bar{C}$ is the weighted mean of $1/C$. The weight for plants of the same shape intersected on the line is proportional to $\Sigma(1/D)$, so that $1/\bar{C}$ is approximately equal to the unweighted mean of $1/C$ taken over all plants of the species in the plane.

The percentage cover by plants of a particular shape is estimated by

$$\frac{100B\Sigma D}{nL}. \tag{5}$$

The factor of proportionality, B, is the ratio of the weighted mean chord on the intercept to the weighted longest chord, i.e. the ratio of

$$\frac{\pi A_0}{\int_0^\pi W_0 d\theta} \text{ to } \frac{\int_0^\pi D_0 W_0 d\theta}{\int_0^\pi W_0 d\theta} \text{ or } B = \frac{\pi A_0}{\int_0^\pi D_0 W_0 d\theta}.$$

Taken over a species the percentage ground cover is estimated by $\frac{100\Sigma(B\Sigma D)}{nL}$, which may be written $\frac{100\bar{B}\Sigma\Sigma D}{nL}$, where $\bar{B}$ is the weighted mean of B, the weight for each shape of plant being proportional to ΣD. An approximate value of $\bar{B}$ is the weighted mean of B of all plants of the species in the plane, weighting the individual values of B proportionally to the area of the plant. For an ellipse with semi major and semi minor axes of a and b respectively and eccentricity e

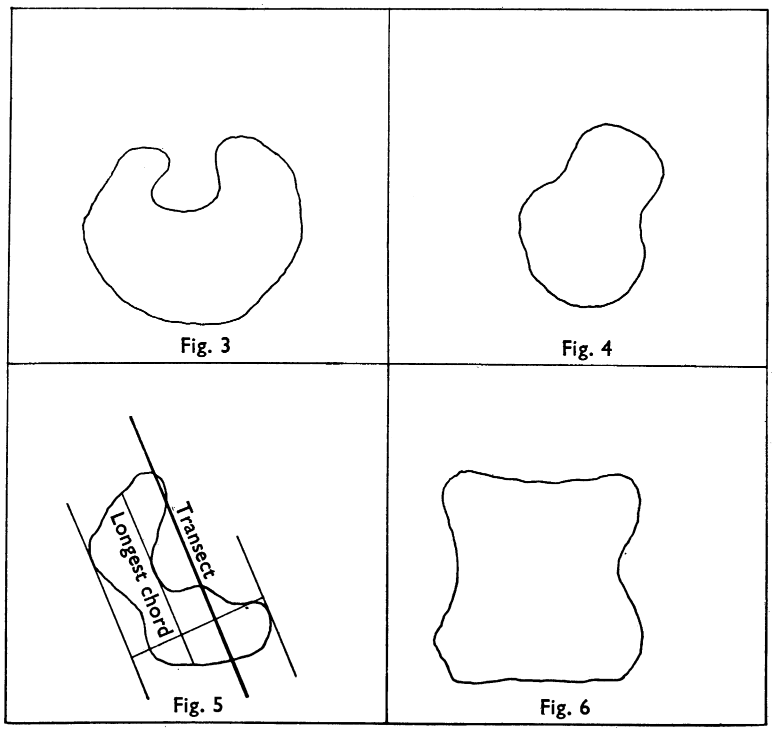
$$D = \frac{2ab}{(a^2 \sin^2\theta + b^2\cos^2\theta)^{\frac{1}{2}}}, \quad W = 2(a^2\sin^2\theta + b^2\cos^2\theta)^{\frac{1}{2}},$$

$$C = \frac{1 - \frac{1}{2}e^2}{(1-e^2)^{\frac{1}{2}}}, \quad B = \tfrac{1}{4}\pi.$$

The following table gives values of C and B for ellipses of various eccentricities and irregular shapes of the forms given in Figs. 3–6.

Table 1

Shape	C	B
Ellipse ($b/a=1$)	1·000	0·785
($b/a=0·8$)	1·025	0·785
($b/a=0·6$)	1·134	0·785
($b/a=0·4$)	1·450	0·785
Irregular (Fig. 3)	1·022	0·676
(Fig. 4)	1·064	0·739
(Fig. 5)	1·338	0·538
(Fig. 6)	1·090	0·675



Figs. 3–6. Irregular closed shapes (see text).

The conclusion would seem to be that moderate distortion as represented by an ellipse with $b/a=0·8$, and by Figs. 3, 4, 6, will cause an overestimate of about 5 per cent if the formula $\frac{\Sigma(1/D)}{nL}$ is used for density, and the percentage cover will also be overestimated if $\frac{100\pi\Sigma D}{4nL}$, which is appropriate for circles and ellipses, is used. These examples suggest that a fair estimate for most plant material would be given by $\frac{\Sigma(1/D)}{1·05nL}$ for density and $\frac{75\Sigma D}{nL}$ for percentage cover.

IV. Estimation from transect chords with supplementary data on longest chords

The expression $\dfrac{\Sigma(1/D)}{\bar{C}nL}$ may be written as

$$\frac{m^2}{\bar{C}nL\Sigma D}\times\frac{\text{A.M.}}{\text{H.M.}} \quad \text{or} \quad \frac{\bar{B}m^2}{\bar{C}nL\Sigma Ch}\times\frac{\text{A.M.}}{\text{H.M.}}, \tag{6}$$

where A.M. and H.M. are the arithmetic and harmonic means respectively of longest chords parallel to the transect of plants intersected by the transects, and m is the number of plants intersected, excluding plants with part chords at one end of the transect. That is, if the longest chords of a random subsample of the plants intersected by the transects are measured and the ratio of arithmetic to harmonic mean computed, this could be used as a correction factor to a crude estimate of density given by $\dfrac{\bar{B}m^2}{\bar{C}nL\Sigma D}$.

The usefulness of (5) and (6) would in practice seem to be limited to plant forms with fairly shallow serrated edges. Where on the one hand the serrations are deep and irregularly spaced the error in estimating B and C from a few representative shapes would probably be quite serious. On the other hand, where the plant boundaries are relatively smooth, bias introduced by the use of reciprocals of chords intersected on the transect may be negligible. With ellipses an unbiased estimate is given by

$$\frac{(2/\pi)\,\Sigma(1/Ch)}{\bar{C}nL}, \tag{7}$$

since the expectation of $2/\pi Ch$ is $1/D$, and the error in applying this to other oval shapes of small eccentricity could be safely ignored.

V. Experimental application of methods

To illustrate the application of the various expressions an artificial population of closed figures (Fig. 7) representing plants was prepared on a sheet of ruled paper 24 × 36 in. Care was taken in drawing them to avoid any consistency in the direction of greatest chords. The area to be sampled was the central 20 × 32 in. The boundary of the area sampled was divided into 4 in. lengths and plants lying on the boundary were considered to be within or outside the area in alternate lengths. The excluded plants have been hatched.

The positions of fifty line transects taken parallel to the length of the sampled area and restricted for convenience of measurement to rulings 1/10 in. apart were determined by random numbers. Plants excluded along the upper and lower boundaries were ignored in making the transect measurements, while for those plants which were accepted and intersected by the transects, the transect was projected to complete the chord. Because opposite sections on the upper and lower boundaries were not both sections of acceptance or rejection, a plant on the boundary at one of the ends of any transect was accepted and at the other end rejected. Plants on the side boundaries intersected by the line quadrats were accepted even if hatched, otherwise a bias would be introduced.

The sample model could have been simplified by eliminating all plants lying on the boundary, but it was thought desirable to deal with a more natural situation. However, in field sampling of vegetation by ecologists, boundaries would not usually be

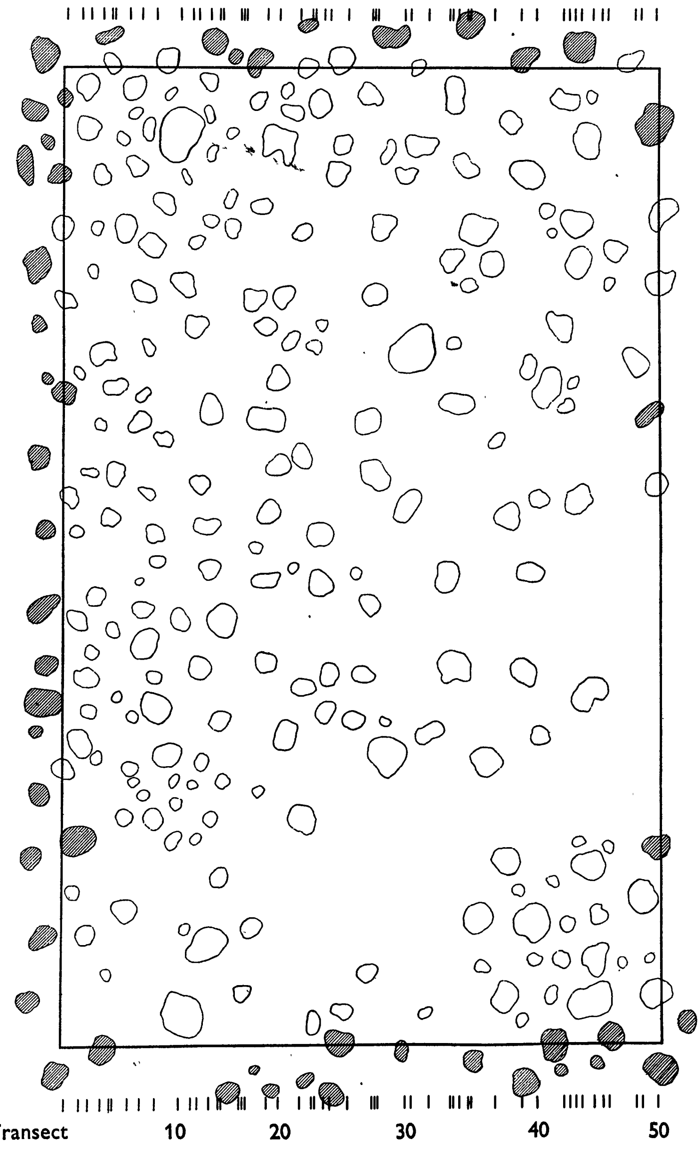


Fig. 7. Reproduction of an artificial population of closed figures showing the position of fifty transects.

exactly defined and problems of securing unbiased estimates within a defined boundary using line transects would not explicitly arise. It was necessary, of course, to use a bounded area for illustration in order to make a comparison between actual and estimated values.

The following data are given in units of length of 1/10 inch and in units of area of 1/100 square inch.

Total sampled area	64,000
Total length of transect	16,000
Number of accepted plants	213
Measured area of accepted plants	8,760
Total length of chords of intersection	2,237
Total length of longest chords	2,939
Sum of reciprocals of chords of intersection	90·8193
Sum of reciprocals of longest chords	57·5254
Total number of plants intersected	377

For estimates of density $\bar{C}$ and $\bar{B}$ were taken to be 1·05 and 0·75 respectively. The data incidentally provide a sample estimate of $\bar{B}$ of $\frac{2237}{2939}$, or 0·7611.

The estimate of the number of plants for an area of 64,000 units using (4) is

$$\frac{57{\cdot}5254 \times 64000}{16000 \times 1{\cdot}05} = 219, \text{ S.E.} = 14.$$

The standard error is based on the variation in the estimate from the separate line transects. The subjective error in the estimate of $\bar{C}$ has been ignored, as well as a bias towards overestimating the standard error appropriate to the mean by about 10 per cent due to ignoring finite sampling of 50 out of 240 strata into which the area was artificially divided. The value for A.M./H.M. for all plants intercepted is 1·189; for a random sample of five transects (Nos. 7, 28, 29, 40, 50) the ratio is 1·116. The estimate of density using (6) is

$$\frac{0{\cdot}75 \times 64000 \times 377 \times 377 \times 1{\cdot}116}{1{\cdot}05 \times 16000 \times 2237} = 203.$$

The estimate of density using (7) is

$$\frac{2 \times 90{\cdot}8193 \times 64000}{\pi \times 1{\cdot}05 \times 16000} = 220, \text{ S.E.} = 16.$$

The percentage area covered is $\frac{8760 \times 100}{64000} = 13{\cdot}69$. The estimate from chords of intersection is

$$\frac{2237 \times 100}{16000} = 13{\cdot}98, \text{ S.E.} = 0{\cdot}67.$$

The estimate from longest chords is $\frac{2939 \times 0{\cdot}75 \times 100}{16000} = 13{\cdot}77$, S.E. $= 0{\cdot}66$.

VI. Separate estimates of density and plant cover

The foregoing indicates that where the vegetation has a relatively smooth boundary, either in fact or through ignoring minor protuberances in measurement, the density is proportional to the sum of the reciprocals of chords of intersection, the constant of proportion varying slightly with the eccentricity of the average shape. Where the vegetation has a serrated boundary this formula will be biased and the measurement of the longest chord parallel to the transect for all plants or for a supplementary sample could be considered as a means of overcoming the bias. These procedures, however, introduce hazards in estimating the appropriate correction factors to allow for irregularity in shape in estimating density and cover. If small plants or seedlings are present and are to be included in the density study, none of the above methods will be free from bias introduced through necessarily large relative errors in measurements of chords and diameters of these plants. In this case the seedlings would have to be estimated separately on a quadrat basis.

A practical method of obtaining an unbiased estimate of density is to use a strip quadrat consisting of two parallel transects. Chords are measured on one transect. Then a plant count of species in the strip is made adopting the convention that (*a*) a plant is acceptable only if its centre lies in the strip, or (*b*) where centres are difficult to define, as in some old perennial grass clumps, the convention can be adopted to accept a plant which lies wholly in the strip or is cut by one designated side or edge. A plant cutting both edges is accepted if the centre of gravity of its projection lies in the strip. If seedlings are very numerous, they can be counted in subsamples of predetermined length and position within the strip.

VII. Comparison of composite method with quadrat sampling

This combination of methods to secure unbiased estimates of percentage cover and density invites the question as to whether the gain in precision in estimating percentage cover by transects relative to square quadrats is lost by the mechanical difficulties of setting up the strip quadrat and dual recording of density and cover measures. The answer would depend very much on the type of vegetation being studied and the degree of aggregation. Where the strip quadrat can be mounted with ease and there is also high aggregation the advantage without doubt would lie with the composite method for both measures. Otherwise it is unlikely that the gains would be important, especially as cover, density and diameter frequency distribution can be extracted from single observations on plants using square or near-square quadrats. Moreover, where changes in time are required, the changes in particular classes of aggregates may be essential to understand the overall change and for this purpose fixed quadrats of square shape would necessarily be taken.

VIII. Conclusion

A number of alternative procedures have been considered for the estimation of density in conjunction with cover. For relatively homogeneous stands or for studies of change at fixed sites the square quadrat seems to have the advantage. For surveys in heterogeneous stands the transect in conjunction with the strip quadrat will give unbiased and efficient estimates of density and cover. For established perennials in open shrub or grassland communities with plants of fairly regular shape, cover and density of each species can be estimated from the transect chords. With moderately irregular boundaries the estimation of density through the use of supplementary data on longest chords parallel to the transect from a subsample of the plants intersected could be considered, especially where cover is the primary measure and density only auxiliary.

IX. Summary

The use of chords of intersection on line transects to provide an estimate of the number of plants per unit area is examined. Objections to the use of this method in particular circumstances are considered and alternative methods for estimating density are suggested.

REFERENCE

Bauer, H. L. (1943). The statistical analysis of Chaparral and other plant communities by means of transect samples. *Ecology*, **24**, 45–60.